

Ex-Day Behavior When Investors and Professional Traders Assume Reverse Roles: The Case of Danish Lottery Bonds¹

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Lottery bonds are Danish Treasury obligations which make coupon payments by lottery. Professional traders have a tax preference for the coupon lottery, while investors are tax-neutral and take the other side of the trade. Consistent with tax-based explanations of abnormal ex-day returns, we find that prices fall by *more* than the lottery mean. Surprisingly, we also find that the price drop over the lottery decreases with the lottery variance. This suggests that investors do not like the lottery. In fact, the Danish Treasury has been able to sell more lottery bonds only by offering above market interest rates. *Journal of Economic Literature* Classification Numbers: G12, G35, G63. © 2002 Elsevier Science (USA)

Key Words: lottery bonds; ex-dividend day; tax differential; diversifiable risk; downward-sloping demand curve.

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1. INTRODUCTION

A longstanding debate exists in the finance literature concerning the interpretation of the empirical fact that on the ex-dividend day stock prices decrease by less than the amount of the dividend. Initially, Elton and Gruber (1970) argued that the low valuation of the dividend reflects the personal tax disadvantage of dividends versus capital gains. Later, Kalay (1982) and Boyd and Jagannathan (1994) proposed the alternative interpretation that the low valuation of the dividend reflects the cost of arbitrage. Specifically, tax neutral, professional traders should pick up the supply of shares from investors who trade to avoid the dividend tax. Competition among the professionals ensures that the marginal valuation of the dividend is one-for-one, but on average prices fall by *less* than the amount of the dividend to cover the transaction costs.

In this paper, we study the ex-dividend day problem in a different institutional setting to address the question: Are abnormal ex-day returns best interpreted as the tax disadvantage of dividends, the cost of arbitrage, or a blend of the two? This is important in order to understand how frictions and arbitrage influence price formation which ultimately matters to investors who trade for tax reasons, corporations which issue securities, and to governments which lose tax revenue. In our case, investors and professional traders assume reverse roles. Professional traders have a tax preference for the dividend and demand the securities even if prices fall by more than the amount of the dividend. Investors, on the other hand, are tax neutral and willing to sell if prices fall by sufficiently more than the dividend to cover their transaction costs. Consistent with the costly arbitrage model of Kalay (1982) and Boyd and Jagannathan (1994), we find in our data set that the marginal valuation of the dividend is one-for-one, but that prices on average fall by *more* than the amount of the dividend. Hence, we conclude that abnormal ex-day returns reflect the cost of arbitrage.

We study lottery bonds in Denmark, which are Danish Treasury obligations and make coupon payments by lottery. The bonds are numbered and payments are made through random drawings of these numbers. Most bonds receive no payment, while a few winning bonds receive prizes up to 10,000 times the face value (one million kroner).² Hence, the outcome of the coupon lottery is extremely risky from the point of view of a single bondholder, while the total payment is fixed and riskless for the issuer. Surprisingly, we find that the variance of the coupon lottery influences the ex-day returns of the bonds. When the lottery variance is low, prices fall by more than the lottery mean, but when lottery variance is high, prices fall by less.

The empirical results in this paper are intriguing. First, the fact that prices, on average, fall by more than the lottery mean suggests that a few professional traders with nonrepresentative tax preferences significantly influence security prices even if the supply from nonprofessional investors ultimately exhausts their demand

² An approximate exchange rate is 8 DKK/USD (2001).

for the coupon lottery. This supports the intuition of Kalay (1982) that a few professional traders are critical to the price formation around the ex-dividend day. Second, the finding that ex-day returns increase with the variance of the coupon lottery suggests that lottery risk, which is diversifiable by construction, influences securities pricing. The apparent risk aversion in prices suggests that the Danish Treasury has designed securities which the market does not want. In fact, we document that lottery bond prices decrease significantly each time the Danish Treasury issues more lottery bonds. In other words, the market does not perceive the lottery bonds as close substitutes for other financial securities, which means that the demand curve for lottery bonds is strictly downward sloping.

The evidence in our paper is taken from an institutional setting which controls for some of the factors that influence ex-day returns in general. This methodological approach has been taken by several other papers. For example, Barclay (1987) documents that stock prices fell by the full amount of the dividend prior to the introduction of income tax in the United States. Furthermore, Green and Rydqvist (1999) study the ex-day behavior of Swedish lottery bonds, where all agents have a tax-preference for the cash distribution. Consistent with tax-based explanations, they find that average marginal tax rates according to tax returns can be imputed from the price change over the cash distribution. There are also studies which do not support the view that differential taxation and transaction costs are at work. For example, Eades *et al.* (1984) document abnormal returns around stock dividends and stock splits which are tax neutral events, and Frank and Jagannathan (1998) find that stock prices fall by only half the amount of the dividend in Hong Kong where both dividends and capital gains are untaxed. Furthermore, Kato and Loewenstein (1995) report that stock prices increase over the ex-dividend day in Japan due to spurious institutional factors, and Michaely and Murgia (1995) find larger ex-day return than can be motivated by even the highest possible tax penalty on dividends in the Italian stock market.

Some results of our paper are similar to those in the two studies of Swedish lottery bonds by Green and Rydqvist (1997, 1999). The construction of the coupon lottery and the tax treatment of the lottery proceeds are similar in Denmark and Sweden. However, all agents in the Swedish market, both professional traders and nonprofessional investors, have a tax preference for the coupon payment, and Green and Rydqvist (1999) also find that ex-day returns reflect those tax preferences. In Denmark, on the other hand, professional traders and nonprofessional investors have opposing tax incentives, so we do not know a priori whether ex-day returns reflect marginal tax rates, transaction costs, or a blend of the two. Our data seem to suggest that ex-day returns reflect transaction costs rather than marginal tax rates in Denmark. Green and Rydqvist (1997, 1999) also report evidence that the variance of the coupon lottery reduces lottery bond prices.

The rest of the paper is organized as follows: Section 2 summarizes the institutional features of the Danish lottery bond market. Section 3 analyzes the price formation over the lottery. The event study results are presented in Section 4. The paper is concluded in Section 5.

TABLE I
Prize Structure

Prize	Number of prizes	Probability of receipt	Mean	Variance
0	997,643	0.997643	0.00	399
500	2,000	0.002000	1.00	460
1,000	100	0.000100	0.10	96
5,000	80	0.000080	0.40	1,984
10,000	50	0.000050	0.50	4,980
25,000	40	0.000040	1.00	24,960
50,000	30	0.000030	1.50	74,940
100,000	25	0.000025	2.50	249,900
250,000	20	0.000020	5.00	1,249,800
500,000	8	0.000008	4.00	1,999,840
1,000,000	4	0.000004	4.00	3,999,840
Sum equals mean and variance			20.00	7,607,199

Note. Bond issued in 1980 with 1 million bonds outstanding with 200 kroner face value per bond. The lottery is annual. For each prize level, the mean is the probability times the prize, and the variance the probability times the prize minus the mean squared. For the entire lottery, the mean and variance equal the sum across rows. Prizes are quoted before lottery tax.

2. DANISH LOTTERY BONDS

2.1. Primary Market

Lottery bonds are Danish government obligations which make coupon payments by lottery. Each bond entitles its owner to one or two coupon lotteries per year over its lifetime. The bonds are numbered and payments to particular bonds are based on drawings of these numbers. The drawings are without replacement, so a single bond can win at most one prize in each lottery.³ A winning bond is retained by its owner and has an equal chance to win lottery prizes in subsequent lotteries. The total coupon payment is fixed, so the liability of the Danish government is riskless. An example of the prize structure is provided in Table I. This bond has face value 200 kroner. There are one million bonds outstanding of which 2357 bonds win a prize in the annual lottery. Therefore, the probability of winning a prize is 0.2357% and the probability of losing 99.7643%, so the outcome of the coupon lottery is extremely risky from the point of view of a single bondholder. The prizes range from 500 kroner to 1 million. The lottery mean is 20 kroner and the lottery variance 7.6 million.⁴

³ This is not quite true for some bonds. For example, consider bond 1977. There are 1 million bonds of each tranche. The bonds are sorted into 1000 series with order numbers between 1 and 1000. The 2000 smallest prizes are paid by drawing two order numbers which are constant across the 1000 series. The winner of one of the small prizes may also win one of the larger prizes. The coupon lotteries for the bonds issued in 1954 and 1980 are similar, but not for any of the other bonds.

⁴ Prizes and coupon rates in this section are quoted before lottery tax. See Section 2.3.

The Danish government floated seven issues of lottery bonds over the period we study. We shall refer to the various bonds by their issue year. The first bond was issued in 1948 and the last in 1980. The bonds are long-term obligations with between 10 to 25 years to maturity. However, the Danish Government has established a practice to exchange maturing bonds for new bonds with identical face value, coupon lotteries, but new maturity. For example, the bond issued in 1948 was scheduled to mature after ten years. In 1958, the bondholders could choose between cash redemption and receiving a new bond with maturity 10 years later. At the time for the next maturity, in 1968, the repayment of the loan was again postponed by ten years to 1978, then to 1988, and, for the last time, to 1998. On January 12, 1998, the Treasury announced that all lottery bonds will be redeemed at their next scheduled maturity. Before the change of policy, the bonds were effectively perpetuities.

Lottery bonds are issued at par and sold by subscription at post offices, banks, and other financial institutions. Table II summarizes the key bond characteristics. The aggregate issue amount is 1.2 billion kroner. This is divided into tranches of 100 or 200 million kroner per bond. The issue amount for 1977 is 500 million, when the Treasury issued five tranches of 100 million kroner each with separate lottery dates, but identical lotteries and maturity. Each of the five tranches of bond 1977 also carried the right to a small initial lottery with mean 0.50 kroner and variance 51,000. The bonds issued in 1965 and 1969 have identical lotteries but different lottery and maturity dates. Coupon lotteries are held semi-annually except for bond 1980 with an annual lottery. The total number of coupon lotteries is 485, which are spread out over 479 different calendar days. Bonds which have matured and have been exchanged for new bonds are putable and can be sold

TABLE II
Danish Lottery Bonds

Bond	Amount (million)	Face value	Lottery mean	Lottery variance	Number lotteries	Options
1948	100	50	0.875	38,030	45	Put
1954	100	50	1.000	57,205	48	Put
1959	100	50	0.875	49,702	46	Put
1965	100	100	3.500	237,145	48	Put
1969	100	100	3.500	237,145	49	Put, Call
1977	500	100	4.000	1,362,334	225	Call
			0.500	51,000	5	
1980	200	200	20.000	7,607,199	19	Call
Sum	1,200				485	

Note. Issue year, issue amount in million kroner, face value per bond (kroner), lottery mean and variance (kroner before lottery tax), and the number of lotteries in 1976 through March 2000. The first five lotteries of bond 1977 are different from the remaining 225 lotteries of this bond. Lotteries are semi-annual except for 1980 with an annual lottery. Putable bonds can be sold back to the Treasury at face value at any time, and callable bonds can be redeemed with one year notice.

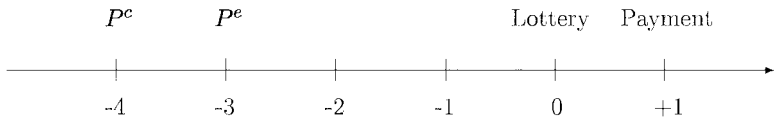


FIG. 1. Time line. The lottery bond is traded cum-lottery at price P^c four business days before the lottery day 0, and ex-lottery at price P^e three business days before. Small prizes are paid the day after the lottery, and large prizes two days after.

back to the Treasury at any time, and the three most recently issued bonds are callable.

2.2. Secondary Market and Data

The secondary market begins when all the bonds of one issue have been sold. Buy and sell orders are passed through brokers to the Copenhagen Stock Exchange. When a bond is offered for sale, the broker collects the bond from the seller. Three business days after the bond has been sold, the broker delivers the bond to the buyer and collects the payment for the seller. The settlement procedure is critical to determine who can claim the proceeds from the coupon lottery. Consider the time line in Fig. 1.

The winning numbers are announced on day 0. The holder of a winning bond collects the prize by physically presenting the bond at a post office or at the Central Bank. If a transaction takes place four business days before the lottery, the settlement is one day before the lottery and the buyer can claim the prize that falls on a winning bond. On the other hand, if the transaction takes place three business days before the lottery, settlement is on the lottery day. In this case, the broker is obliged to present the winning bond to the Central bank and to deliver the prize to the seller. Following the convention from the equity market, we shall refer to day -4 as the last day the bond trades cum-lottery and to day -3 as the first day the bond trades ex-lottery. Small prizes below 5000 kroner are paid the next day, while prizes above 5000 kroner are paid two days after presenting the winning bond.

Before November 1988, buy and sell orders were cleared through a daily auction, and from November 1988, buy and sell orders are matched electronically. For 1976–1988, we collect the best buy limit order, which has not been cleared in the auction, and the auction trading volume. From the electronic system, we retrieve the weighted average transaction price, the trading volume, and the best buy and sell limit orders at the end of the day.⁵

Table III reports statistics on daily trading volume. The bonds with equal face value have similar characteristics and have been grouped. Between 100 and 200 bonds are traded per day and tranche. This corresponds to between 5000 and

⁵ Data for 1976–1984 were collected manually from the official quotation lists of the Copenhagen Stock Exchange, data for 1985–February 1997 were provided by the Aarhus School of Business, and for March 1997–2000 by ALOC Bonnier A/S, which also provided us with buy and sell limit orders from 1990–2000.

TABLE III
Trading Volume

Face/ bond	Number (median)	Kroner (median)	Positive (proportion)	Turnover (annual)	Spread (mean)
50	100	5,000	0.606	0.043	0.028
100	100	10,000	0.625	0.069	0.025
200	200	40,000	0.730	0.135	0.015

Note. The median number of bonds traded per day, the median kroner value traded per day (face), the proportion of days with positive volume, and the annual turnover. The spread is constructed from the mode of the difference between the best buy and the sell limit order in 1990–2000. The reported statistic is the average of the mode divided by the weighted average transaction price.

40,000 kroner face value. There is positive trading volume between 60 and 73% of the business days, and the annual turnover is between 4.3 and 13.5%. These volume statistics demonstrate that this is a really small financial market.

Table III also reports in the rightmost column a measure of transaction costs. For each bond and day in 1990–2000, we compute the difference between the best buy and sell order. The mode equals 2 kroner for the bonds with face value 50 kroner and 5 kroner for the other bonds. We focus on the mode instead of the mean, because buy and sell orders are not always available and, occasionally, when buy and sell orders are available, the difference is very large. The spread in Table III is the time-series and cross-section average of the mode divided by the weighted average transaction price. As can be seen, the average spread falls from 2.8% for the bonds with the lowest face value to 1.5% for the bond with the highest face value. The spread pattern is the flip side of the volume. Transaction costs also include broker commission, which was 0.50% of face value before July 1988 and 0.75% of price after.

Orders for lottery bonds must be separated by 0.25 kroner in the call auction (1976–1988) and by 0.05 kroner in the electronic system (1989–2000). The price ticks are small relative to both the mode of the buy–sell spread (above) and the lottery means (Table II). Therefore, rounding errors caused by price discreteness as suggested by Bali and Hite (1998) for the U.S. equity market should be less problematic in the Danish lottery bond market.

2.3. Tax Environment

Proceeds from the coupon lottery are subject to a special lottery tax at a flat rate of 15% on payments above 200 kroner. The net payment does not contribute to ordinary income, so this tax simply serves to reduce lottery prizes by approximately 15%. The prizes in Table I and the lottery means and variances in Table II are quoted before lottery tax. In what follows, we shall quote prizes net of the lottery tax so that coupon payments are effectively tax-exempt.

Capital gains and losses on lottery bonds are calculated as the difference between the sell price and the purchase price adjusted for broker commission. Lottery bond

prices are quoted with accrued interest, so the price appreciation between two coupon payment dates is treated as capital gain.⁶ Consequently, the price drop over the lottery is treated as capital loss.

The Danish tax code distinguishes between professional traders and nonprofessional investors. Professional traders pay ordinary income tax on capital gains from lottery bonds and can deduct losses which can be carried forward five years. Nonprofessional investors are not taxed on capital gains on lottery bonds, and cannot deduct losses. Hence, nonprofessional investors are tax exempt on income from lottery bonds, while professional traders are exempt from taxation on coupon income, but pay tax on capital gains and can deduct capital losses from ordinary income.⁷

Brokerage firms and commercial banks have professional trader status. Before 1987, the law required that organized security dealers were partnerships with personal liability. Each partner paid personal income tax on his share of the trading profits, so personal income tax applies to 1976–1986. In 1987, the law was abolished and the security dealers incorporated, so the relevant tax code for 1987–2000 relates to corporate income. Personal income tax may be relevant also after 1987, because individuals can get professional trader status depending on how much they trade. Court cases concerning the tax status of individuals suggest their possible participation in the lottery bond market.

The time-series of marginal tax rates is shown in Fig. 2. The upper schedule is the top personal rate which ranges from 63 to 74% over time. The personal income tax table is highly progressive and the top rate kicks in at low income levels. For example, in 2000, the top marginal rate started at the personal income of 267,600 kroner. The lower schedule is the corporate rate which falls between 32 and 50%.

2.4. *Lottery Bond Yields*

We end the institutional description of the Danish lottery bond market by comparing the yields of lottery bonds relative to each other and relative to regular Treasury securities. The evidence in this section suggests that market participants are averse to the lottery variance, which is going to be important in the analysis of ex-day returns below.

A time-series plot of the yield of one lottery bond along with the yield of a regular Treasury security is provided in Fig. 3. The lottery bond yield is the average of the current yields for the two lottery bonds issued in 1965 and 1969. These particular

⁶ With regular bonds, when a transaction occurs between coupon payment dates, the payment for the bond is divided into “accrued interest” and “price.” The accrued interest represents the portion of the coupon the seller has earned by holding the bond through part of the coupon period.

⁷ Two other investor categories are unlikely participants in this market. Pension funds are taxed on coupon income from all bonds including lottery bonds, and foreign investors are taxed on the lottery proceeds in their home countries. Since other investors are tax-exempt, lottery bonds trade at relatively low yields, so pension funds and foreign investors should find lottery bonds unattractive to hold in their portfolios.

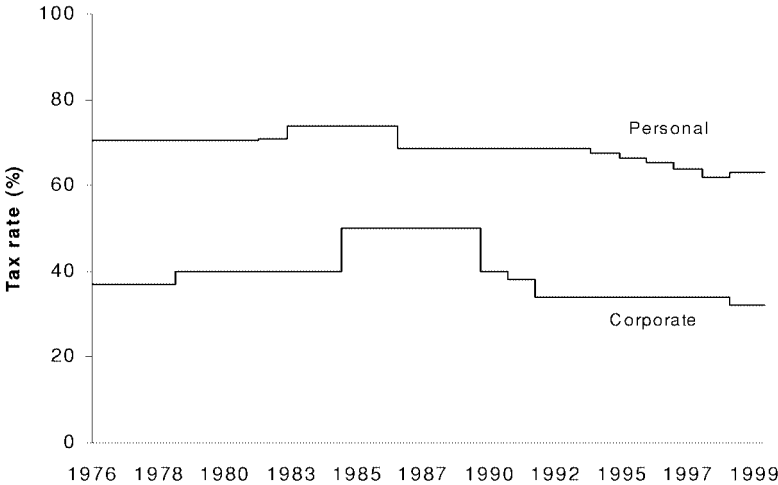


FIG. 2. Marginal tax rates. Top personal and corporate income tax rates.

bonds have been selected because they have been traded throughout the sample period. The current yield is used instead of the yield to maturity because the bonds are best described as perpetuities up to the change of policy to redeem the bonds after 1998. The Treasury yield is constructed from six noncallable, bullet

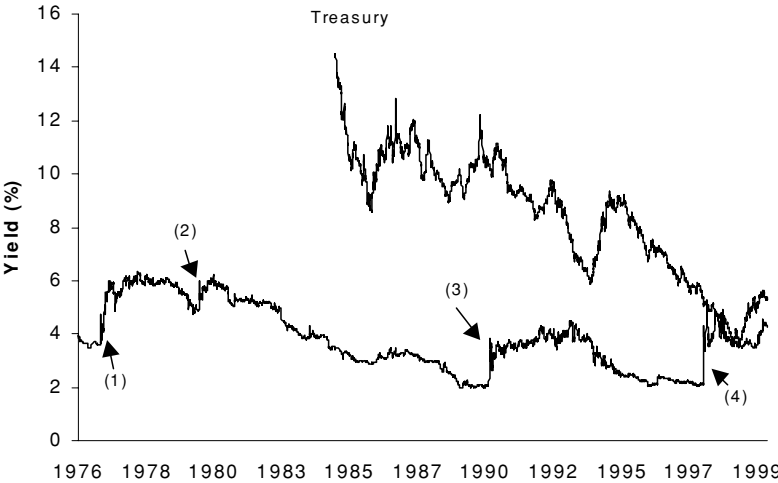


FIG. 3. Current yield. The average current yield for lottery bonds 1965 and 1969. The Treasury yield is constructed from six noncallable, Treasury bonds with between 7 and 11 years to maturity. The time series starts in 1985. The marked announcements are (1) the issuance of bond 1977, (2) issuance of bond 1980, (3) opening of bank accounts with interest paid by lottery, and (4) new policy to redeem all bonds at maturity.

TABLE IV
Significant Price Changes

Event	Description	Current yield old securities	Coupon yield new security	Old securities (−3, +3)	
				ΔYield	Return
(1)	Bond 1977	3.19%	6.92%	+0.46%	−14.47%
(2)	Bond 1980	5.10%	8.54%	+0.29%	−4.26%
(3)	Bank accounts	2.43%	6.30%	+0.95%	−33.57%
(4)	Redemption	2.39%	n.a.	+2.12%	−45.43%

Note. Announcement of the issuance of bond 1977, bond 1980, the opening of bank accounts with interest paid by lottery, and the change of Treasury policy to redeem outstanding bonds at the next maturity. Average current yield of outstanding lottery bonds before the announcement, the coupon yield of the new security, the change in the current yield of old securities from three days before the announcement to three days after, and the average cumulative return over the six day period.

bonds with between 7 and 11 years to maturity. The time series starts in 1985 when interest rate data become electronically available.

The plot shows that lottery bond yields are substantially lower than regular Treasury yields most of the time. There are many possible reasons. Lottery bonds make tax-free coupon payments, while payments from regular Treasury securities are subject to high marginal tax rates. Furthermore, the lottery bonds are putable, bond 1965 throughout the sample period and bond 1969 since 1994, which caps the current yield of those bonds to 6.36%. Finally, lottery bonds are in demand for money laundering, because the holder of a large portfolio of bonds must only reveal the ownership of the few winning bonds, in order to claim the proceeds from the lottery, but can effectively hide the ownership of all the nonwinning bonds.

Four events are marked in Fig. 3, and descriptive statistics related to those events are provided in Table IV. We start with the most recent event on 12 January 1998 marked (4) in the figure. On this day, the Treasury announced the change of policy to redeem all lottery bonds at maturity. The market responded by raising the current yield by more than 2 percentage points. Prices fell by 45% from an average price of 250% of par down to 112%. As suggested by the high price level, the lottery bonds traded at current yields significantly below their coupon yields, so market prices fell as the bonds were transformed from perpetuities to bonds with finite maturity.⁸

The other three events are supply increases. In 1976, the Treasury announced the issuance of bond 1977, which doubled the outstanding amount of lottery bonds from 500 to 1000 million kroner. In 1980, the Treasury announced the sale of

⁸ Consistent with the perpetuity interpretation, lottery bond prices did not change noticeably in the past when the exchange offers were announced. During our sample period, old bond 1948 was exchanged for a new bond in 1977 and again in 1988, bond 1954 in 1984, bond 1959 in 1987, bond 1965 in 1995, and bond 1969 in 1994. The change in the average current yield, across the six events, measured over the six-day period around each announcement, is negligible +0.038%.

600 million kroner of face value of bond 1980.⁹ Finally, in 1990, the commercial banks were granted the right to offer bank accounts with interest paid by lottery.¹⁰ Subsequently, the market value of the bank accounts rose to approximately 9 billion kroner compared to the face value of the lottery bonds of 1.2 billion kroner. As can be seen in Table IV, in all three cases, the coupon yield of the new security was set much higher than the current yield of traded securities. As a result, current yields of traded securities increased and bond prices fell dramatically.

The negative market reaction to the three supply increases suggests that the demand for lottery bonds is inelastic. Inelastic demand curves are in sharp contrast with standard asset pricing theory which explains that financial securities are perfect substitutes for each other and implies that demand curves for individual securities are horizontal. The natural candidate for the inelasticity is the lottery feature, which market participants seem to be unable to hedge and, instead, require a higher expected return to buy more bonds. This interpretation is supported by the fact that the new securities are offered to the market at higher yields than existing securities. Despite the high interest rates on the new securities, the new lottery bonds were not easy to sell. A newspaper article mentions that the placement period for the fifth tranche of 1977 lasted longer than expected, and the placement of bond 1980 actually failed, since only 200 million of the originally planned 600 million was placed.

Next, we compare the yields of the lottery bonds against each other. Summary statistics of the current yields are provided in Table V. We focus on the period when all bonds were traded between 25 March 1980 when bond 1980 started trading and 12 January 1998 when the Treasury announced its new policy. A clear pattern emerges. The three oldest bonds in the top of the table have much lower average yields than the bonds with higher face value, and the most recently issued bond in 1980 has the highest average yield by far. This pattern is clearly correlated with the pattern of lottery variance in Table II. We interpret this correlation as additional evidence that investors do not like the lottery.¹¹

⁹ The intention to issue the new lottery bond 1977 was announced on 6 October 1976, and the terms on 26 November 1976. For 1980, the intention to issue was announced on 8 April 1980, and the terms on 8 May 1980. We report in Table IV the market reaction to the announcement of the terms of the new issue, since the announcement of the intention to issue goes together with several other Government proposals related to the recession. However, average lottery bond yields increased also at the first announcement by 0.62 and 0.47%, respectively.

¹⁰ The possibility of bank accounts with interest paid by lottery was first proposed on 17 January 1990. The proposal was accepted by the Parliament on 8 May 1990, and the bank accounts opened on 1 July 1990. The market reaction took place about 10 days before the opening of the accounts as the banks increased their marketing activities, but we have no precise event to tie the market reaction to. The bank accounts paid 5.1% by lottery and 4% by fixed interest subject to personal income tax. The coupon yield in Table IV assumes that the fixed interest rate is taxed at the 70% personal rate.

¹¹ The time series rarely intersect. The yield series of bond 1980 with the extreme lottery variance does not overlap with any of the other bonds. Bond 1977 with the second highest variance overlaps 22% of the time with bonds 1965 and 1969 with the third highest variance, but never with bonds 1948 and 1959 with the lowest variance. Bond 1965 and 1969 overlap 3% of the time with bond 1954, but never with bonds 1948 and 1959 with the lowest variance. Since lottery bond prices are well above par

TABLE V
Current Yields

Bond	Mean	Min	Max
1948	2.04	1.34	3.05
1954	2.34	1.57	3.54
1959	2.03	1.31	3.04
1965	3.34	1.96	5.66
1969	3.36	1.93	5.86
1977	3.53	2.00	6.55
1980	5.55	3.84	9.51

Note. Percentage current yield over the period 25 March 1981 to 12 January 1998 when all bonds are traded.

Finally, we show a time-series plot of the yield to maturity in Fig. 4 for the two bonds issued in 1977 and 1980 between 13 January 1998 and 7 April 2000. Only for these two bonds, and during this time period, is yield to maturity properly defined. We can see that the yield to maturity is persistently higher for bond 1980 with the higher lottery variance. We can also see that bond 1977 is traded at negative yields to maturity during an extended time period in 1998–1999. Green and Rydqvist (1999) report that Swedish lottery bonds sometimes trade at negative yields and provide a tax-based explanation. We shall return to this issue in Section 4.3 after we have examined our empirical results.

3. PRICE FORMATION OVER THE LOTTERY

In this section, we analyze how taxes and the characteristics of the coupon lottery influence ex-day returns.

3.1. Taxes

We make the following assumptions: (i) Agents are risk neutral. (ii) Short sales are not allowed. (iii) Discounting over the two business days that elapse from the cash settlement of the last trade cum-lottery to the payment of the proceeds is ignored. (iv) Taxes are paid at the time of the trade. The first assumption focusses the analysis on the tax effect. Risk aversion is considered in the next subsection. The no short sale assumption is in line with market practice, and the two final assumptions reduce notation.

The analysis is carried out in three steps. First, we show that professional traders have a tax incentive to generate a capital loss which exceeds the lottery mean. Then, we show that nonprofessional investors are willing to supply their bonds to the

most of the time, and the implicit Treasury policy before 1998 has been to never redeem outstanding bonds, the cross-section variation in put and call options cannot explain the cross-section variation of lottery bond yields.

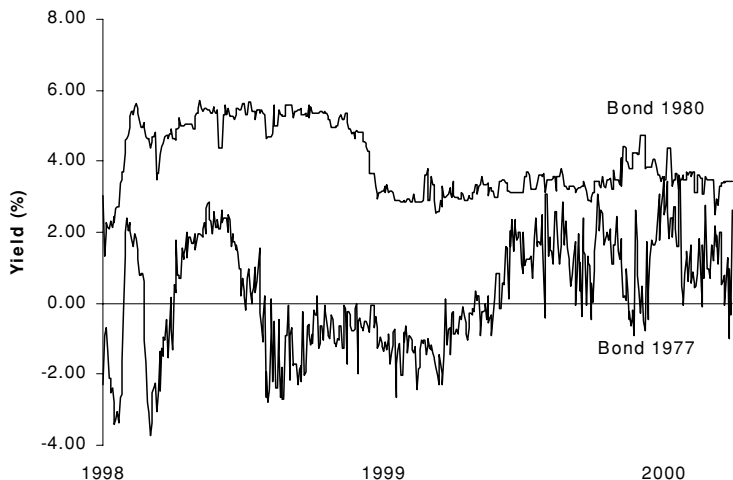


FIG. 4. Yield to maturity. From 13 January 1998 to 7 April 2000 for the lottery bonds issued in 1977 and 1980. The bonds mature in 2002 and 2005, respectively.

professional traders to avoid the capital loss as long as transaction costs are not too high. Finally, we determine the equilibrium prices.

Consider a professional trader with flat marginal tax rate τ which is positive when he has taxable income and zero otherwise. The professional trader faces a transaction-specific cost c , which increases with the trading volume as more time must be devoted to trading in lottery bonds.¹² Suppose that the professional trader purchases the bond cum-lottery at price P^c and sells the bond ex-lottery at price P^e . The expected cash flows from the round-trip transaction are

$$-P^c + P^e + E(C) - 2c - \tau(P^e - P^c - 2c),$$

where $E(C)$ is the lottery mean. The professional trader breaks even when this expression equals zero. After rearranging,

$$P^c - P^e = -2c + \frac{E(C)}{1 - \tau}. \quad (1)$$

The left-hand side is the maximum capital loss the professional trader is willing to bear in return for the lottery. Hence, it is the highest price the professional trader is willing to pay for the coupon lottery. For $\tau > 0$ and c sufficiently small, this amount exceeds the lottery mean, because the capital loss shields ordinary income tax.

¹² For simplicity, we do not distinguish between brokers and other professional traders. For professional traders, who are brokers, the commission equals the expected costs (wages, office, etc.) However, the commission may not cover the additional costs associated with increased trading activity around the lottery. Nonbroker, professional traders must pay the broker commission as an additional cost besides wages, office, etc.

Next, consider a nonprofessional investor who does not pay tax and has no interest in a capital loss which exceeds the lottery mean. Rather, if prices even partially reflect the marginal tax rate of the professional trader, the nonprofessional investor can make a tax-free capital gain by selling the bonds cum-lottery and buying back afterward. The expected cash flows from the reverse round-trip transactions are

$$P^c - P^e - E(C) - 2c.$$

The nonprofessional investor breaks even when this expression equals zero:

$$P^c - P^e = 2c + E(C). \quad (2)$$

The left-hand side is the smallest capital gain the nonprofessional investor requires to carry out the reverse round-trip and sell the coupon lottery to the professional trader. For $c > 0$, this amount exceeds the lottery mean, because the seller demands compensation for the transaction costs.

Trade is carried out if the professional trader values the lottery higher than the nonprofessional investor, which occurs when the left-hand side of (1) exceeds that of (2). We shall allow for the possibility that prices fall anywhere between (1) and (2). Define $\gamma \in [0, 1]$ as the seller's share of the surplus, such that $\gamma = 1$ if the nonprofessional investor gets the entire surplus as in (1) and $\gamma = 0$ if the professional trader gets the surplus as in (2). Then, the actual price change over the lottery is a weighted average of (1) and (2):

$$P^c - P^e = 4c \left(\frac{1}{2} - \gamma \right) + \left(1 + \frac{\gamma\tau}{1-\tau} \right) E(C). \quad (3)$$

In the derivation of this equation, for simplicity, we have assumed that the marginal transaction costs are equal for the professional trader and the investor. The range of possible equilibria is illustrated in Fig. 5. The steep line represents the most the professional trader is willing to pay for the lottery. Its slope is steep because the marginal tax rate is high (see Fig. 2). The flatter line has a slope equal to one and represents the least the nonprofessional investor requires to give up the lottery. It is shifted up from the origin by the transaction costs. Equilibrium prices may fall anywhere between the two lines to the right of the intersection.

Consider the two extreme cases. First, suppose that the professional trader gets the surplus ($\gamma = 0$), which implies that (3) simplifies to (2). This means that there is sufficient supply from nonprofessional investors to exhaust the demand for tax-preferred coupon payments, which means that the marginal tax rate is zero. Prices fall, on average, by more than the lottery mean, but the marginal valuation equals one, i.e., price changes plot along the flatter line to the right of the intersection in Fig. 5. This is the kind of equilibrium envisioned by Kalay (1982) and by Boyd and Jagannathan (1994) for the U.S. equity market.

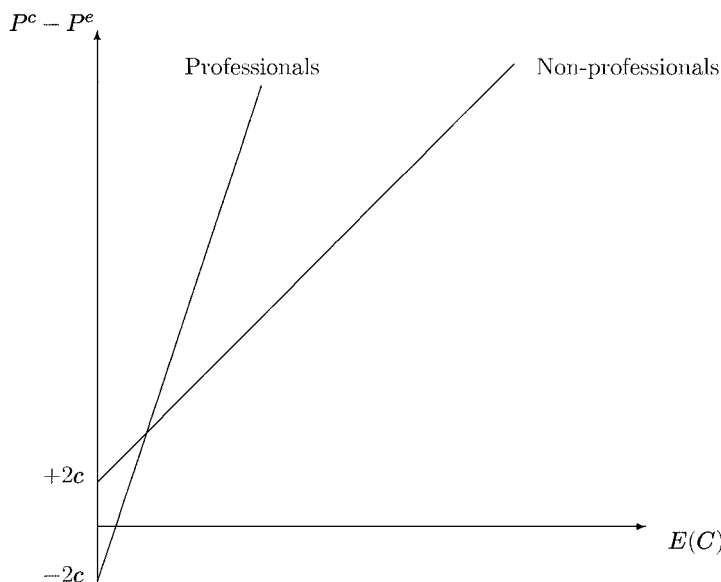


FIG. 5. Trade region. The most the professional trader is willing to pay for the lottery and the least the nonprofessional investor requires to sell the lottery. The area between the lines to the right of the intersection is the gain to trade.

Instead, suppose that the nonprofessional investor gets the surplus ($\gamma = 1$), which implies that (3) simplifies to (1). In this case, the professional traders compete away the profit from the tax arbitrage. Prices fall on average by more than the lottery mean, but, unlike the other equilibrium, the marginal valuation *exceeds* one. This equilibrium requires that the supply of bonds from nonprofessional investors is limited which, in turn, requires that transaction costs are high. Possibly, transaction costs are so high that the only bonds which are offered for sale are supplied by nonprofessional investors who have decided to sell regardless of the lottery and, therefore, consider the transaction cost as sunk ($c = 0$). These are regular trades and the transaction costs for the professional traders are covered by the broker commission. In Fig. 5, the line representing the nonprofessional investors shifts down to the origin, while that of the professional traders shifts up. Price changes plot along the steep line. This equilibrium has the spirit of Elton's and Gruber's (1970) long-term investor, who has decided to trade and chooses between trading before or after the cash distribution. Given the expected capital loss over the lottery, those who have decided to sell will accelerate their sales and supply their bonds cum-lottery, and those who have decided to buy will postpone their purchases to after the lottery. However, with $\gamma = 1$, these timing effects are too weak to move prices away from (1).

In the empirical analysis, we shall test the two extreme models against each other. Both models explain that prices, on average, fall by more than the lottery

mean. However, if nonprofessional investors set prices ($\gamma = 0$), the marginal valuation of the coupon lottery is one, while if the professional traders set prices ($\gamma = 1$), the marginal valuation exceeds one. This feature distinguishes the two models.

A priori, $\gamma = 0$ seems likely because marginal tax rates are high in Denmark, so transaction costs must be very high to deter short-term supply. In addition, the sheer number of nonprofessional investors, who probably hold more than 99% of the bonds, suggests that tax effects should be eliminated from prices. On the other hand, we cannot rule out $\gamma = 1$, or any intermediate split of the gain to trade, for at least two reasons. This is a retail market where most nonprofessional investors might be unaware of the trading opportunity and, therefore, never sell. Some nonprofessional investors may be sophisticated, but they must take into account that systematically selling their bonds over the lottery may alter their nonprofessional tax status. As indicated by the court cases concerning tax status, this could be unattractive. For example, professional traders must pay capital gains tax on equity, while nonprofessional investors have tax exemptions or lower taxation on long-term gains. The *endogeneity* of the tax status rules out pure arbitrage by a few sophisticated, nonprofessional investors, so if we find in the data that $\gamma = 0$ this might be the outcome of the collective effort of many nonprofessional investors.

3.2. Lottery Risk

The tax arbitrage of the professional traders means substituting a certain capital loss for the uncertain income stream from the coupon lottery. In this section, we analyze how lottery risk mitigates the tax arbitrage.

The coupon lottery has no aggregate risk, so the lottery risk is diversifiable by construction. Perfect diversification could be achieved by forming a mutual fund which purchases all the lottery bonds and redistributes the coupon income as risk free claims. No such funds exist, perhaps, since this would be against the issuer's policy to disperse the bonds among retail investors. Less than perfect diversification can be achieved by purchasing many bonds. Diversification is extremely effective since the payments to each bond in a given issue are almost *i.i.d.*¹³ As a result, the variance per unit invested in a portfolio of 100 bonds is only about 1% of the variance per unit invested in a single bond. The lottery risk could matter anyway, because even a large portfolio of bonds will be exposed to the variance of the few large prizes. For example, consider bond 1980 in Table I above. The four top prize levels contribute with about 75% of the lottery mean, but the probability of receiving one of those payments is only 0.000057, so even a portfolio as large as 10,000 bonds would be exposed to considerable lottery variance with respect to 75% of the mean. Furthermore, given that only 200 bonds are traded per day (see Table III), it may be difficult to form a portfolio of as many as 10,000 bonds over a short time period around the lottery.

¹³ The prizes in the lotteries are drawn without replacement. This induces small levels of negative correlation which enhances the benefits of diversification.

A simple way to add lottery risk into the analysis is to substitute the lottery mean in (3) for a certainty equivalent such that the equilibrium condition becomes

$$P^c - P^e = 4c \left(\frac{1}{2} - \gamma \right) + \left(1 + \frac{\gamma \tau}{1 - \tau} \right) [E(C) - \lambda V(C)], \quad (4)$$

where $E(C)$ is the lottery mean, $\lambda \geq 0$ is a risk premium, and $V(C)$ is the lottery variance. The linear specification conforms naturally with the regression analysis in the next section. This equation can be regarded as a special case of the model in Michaely *et al.* (1996, Eq. (7), p. 347). They consider investors with constant absolute risk aversion, differential tax rates and transaction costs and show that the price change over the ex-day is a weighted average of tax rates and transaction costs minus a risk premium for the price uncertainty associated with holding an unbalanced portfolio over the ex-day. Since we cannot measure the distributions of weights, transaction costs, and tax rates in any meaningful way, any observed γ would be consistent with their model.¹⁴

4. EVENT STUDY

We shall first report evidence on abnormal trading volume around the lottery and then abnormal returns over the ex-lottery day.

4.1. Abnormal Volume

We have argued that nonprofessional investors sell their bonds over the coupon lottery to the professional traders who can exploit the ex-day price drop to shield ordinary income tax. This suggests that there is abnormal trading volume around the lottery.

Several papers report evidence of abnormal trading volume around the ex-dividend day in the U.S. equity market (e.g., Lakonishok and Vermaelen (1986), Karpoff and Walkling (1988), Michaely and Vila (1996)). To cope with problems that arise from thin trading, we develop a slightly different methodology. Specifically, a period of 20 business days around the lottery (day -10 to day $+10$) is designated as the event period, and another 20-day period which goes from day -30 to -21 and from day $+21$ to $+30$, is designated as the benchmark period. We pool the lotteries of all bonds with equal face value and estimate for each group of bonds the average daily trading volume across the benchmark periods and the event periods, respectively. The difference between the benchmark and the event period averages is a measure of abnormal trading volume.

¹⁴ Their model is designed to explain trading volume around the ex-day for which testable propositions are generated. An attempt to estimate ex-day returns as a weighted average of marginal tax rates and transaction costs can be found in Michaely and Murgia (1995) using Italian stock price data.

TABLE VI
Abnormal Volume

Face/ bond	Average daily volume			Proportion days with trade		
	Benchmark	Event period	<i>t</i> stat.	Benchmark	Event period	<i>t</i> stat.
50	15.5	22.2	7.7	0.585	0.694	8.5
100	28.2	34.8	4.1	0.616	0.692	10.5
200	143.2	175.7	1.5	0.744	0.788	1.4

Note. Average daily trading volume (thousand kroner) across bonds with equal face value and over time. The event period is defined as the 20 business days around the ex-lottery day, and the benchmark period goes from day -30 to day -21 and from day +21 to day +30. There is one benchmark period and one event period around each lottery. There are 139 lotteries of bonds with face value 50, 327 lotteries of face value 100, and 19 of face value 200. The *t* statistic tests the difference between means.

The estimated means and a *t* test of the difference between means are provided in the middle section of Table VI. Consider first the bonds with face value 50 kroner. The average daily trading volume in the benchmark period is 15,500 kroner, while that in the event period 22,200 kroner. Both numbers are really small in absolute terms, but they are statistically different from each other. The other bonds also have higher trading volume in the event period, but the difference is not statistically significant for the bonds with face value 200 kroner. Since volume data are highly skewed, we also provide a nonparametric test of abnormal volume in the right section of Table VI. As can be seen, the proportion of days with positive trading volume is significantly higher except for the bond with the highest face value. Taken together, the evidence suggests that short-term trading takes place around the lottery.

4.2. Ex-Day Returns

Ex-day returns are computed using buy limit orders from 1976 to 1988 and weighted average transaction prices from 1989 to 2000. When price data are missing for the last day before the bond goes ex, we search backward for the next available price, and when data are missing for the ex-day, we search forward. This procedure induces a positive bias in the estimated ex-day returns. If we find negative ex-day returns despite the bias, this is reassuring of the analysis in Section 3.1 that professional traders have a tax preference for the coupon lottery.

We follow the literature and compute the premium as the price change over the lottery mean,

$$Pr = \frac{P^c - P^e}{E(C)},$$

and the ex-day return

$$r = \frac{P^e + E(C) - P^c}{P^c}.$$

TABLE VII
Descriptive Analysis

	Premium		Return	
	mean	s.e.	mean	s.e.
All	1.461	0.107	−0.0057	0.0015
1948	2.812	0.582	−0.0194	0.0061
1954	2.121	0.293	−0.0152	0.0038
1959	1.998	0.465	−0.0125	0.0053
1965	1.268	0.263	−0.0059	0.0043
1969	1.211	0.415	−0.0038	0.0068
1977	1.116	0.110	−0.0023	0.0018
1980	0.592	0.124	0.0219	0.0064

Note. The premium is the price change over the lottery mean, and the return is the price change plus the lottery mean over the cum-lottery price. There are 485 observations; s.e. denotes standard error.

The means and standard errors of these variables are reported in Table VII. The mean premium across all bonds is 1.461, which is statistically different from one, and the corresponding mean return is −0.57%, which is significantly below zero. The means are not equal across the bonds. The mean premium is significantly larger than 1 for the bonds with face value 50 kroner (1948, 1954, and 1959), and it is significantly less than 1 for the bond with face value 200 kroner (1980). The corresponding returns are significantly negative for the bonds with a premium significantly above 1 and significantly positive for the bond with the premium below 1.

Two results in Table VII indicate that investors do not like the lottery risk. First, the premium and the return across bonds is correlated with the lottery variance (Table II). For example, the bond with the lowest lottery variance (1948) has the highest premium, while the bond with the highest lottery variance (1980) has the lowest premium. Second, the premium for bond 1980 is significantly below one. This finding is inconsistent with (3), which has been derived under risk neutrality and requires a premium above one. These results reinforce our conclusion from Section 2.4 that risk aversion is responsible for the negative price reactions to supply increases and the cross-section variation of lottery bond yields (Table V).

4.3. Structural Estimation

Equation (4) can be estimated with ordinary least squares following the methodology in Boyd and Jagannathan (1994),

$$\frac{P^c - P^e}{P^c} = a + b \cdot \frac{E(C)}{P^c} + c \cdot \frac{V(C)}{P^c} + \epsilon, \tag{5}$$

TABLE VIII
Structural Analysis

Regression	Intercept <i>a</i>	<i>E</i> (<i>C</i>) <i>b</i>	<i>V</i> (<i>C</i>) <i>c</i>	<i>R</i> ²
(1)	0.0097 (0.0037)	1.180 (0.305)	−0.202 (0.078)	0.037
(2)	0.0152 (0.0030)	0.484 (0.143)	—	0.023

Note. The price change is regressed on the lottery mean and variance. All variables are scaled by the cum-lottery price. The coefficient of the variance is multiplied by 10,000. The parameters are estimated with ordinary least squares. Standard errors are reported below. The number of observations is 485.

where

$$\begin{cases} a = \frac{4c(1/2 - \gamma)}{P^c} \\ b = 1 + \frac{\gamma\tau}{1 - \tau} \geq 1 \\ c = -\lambda b \leq 0. \end{cases}$$

The intercept can be either positive or negative depending on the split parameter γ . The results are reported in Table VIII. As can be seen in the top, the intercept is significantly positive, the slope coefficient of the lottery mean is about one, and the slope coefficient of the lottery variance is significantly negative. These parameters are consistent with the equilibrium condition (4). The bottom regression shows that controlling for lottery risk is critical. Otherwise, the intercept remains significantly positive, but the slope coefficient of the lottery mean falls below one.

The estimates in the top row of Table VIII are consistent with the hypothesis that nonprofessional investors exhaust the demand for the coupon lottery ($\gamma = 0$), but they are inconsistent with the alternative hypothesis that professional traders compete away the profit from the tax arbitrage ($\gamma = 1$). We can also reject the hypothesis that the gain to trade is split equally ($\gamma = 1/2$). We base these conclusions on the significantly positive intercept. For $\gamma = 0$, the round-trip transaction cost can be imputed to $2c/P^c = 0.97\%$, which is of the same order of magnitude as the double commission (0.72% pre-1988 and 1.50% post-1988).¹⁵ If $\gamma = 0$, the marginal tax rate is zero. This means that the demand for tax shields in the lottery bond market is exhausted, but not necessarily that demand for other tax shields is exhausted. The aggregate coupon payments for all lottery bonds in one year equals about 70 million kroner, while the taxable income of all commercial banks and brokerage houses in a year is several times larger. Transaction costs and lottery risk may explain why professional traders do not purchase additional tax shelters in the lottery bond market.

¹⁵ The estimate of the round-trip transaction cost is smaller than the sum of double commission and the buy-sell spread reported in Table III. The finding of abnormal trading volume around the lottery suggests, however, that effective spreads are significantly smaller and may be close to zero.

TABLE IX
Institutional Changes

	Before	After	<i>t</i> test
All	1.798	1.135	3.1
1948	3.477	2.053	
1954	2.239	2.003	
1959	3.184	0.704	
1965	1.435	1.102	
1969	1.034	1.395	
1977	1.359	0.892	
1980	0.657	0.555	

Note. The average price change over the lottery mean before and after 20 November 1988. The *t* statistic tests the difference between means. There are 253 and 232 observations in each subperiod.

The standard errors are large, because there is little cross-section variation in the lottery mean. The 485 observations can be grouped into four levels: (i) 5 observations with mean 0.5% (5 initial lotteries of 1977), (ii) 139 observations with mean about 1.5% (bonds with face value 50), (iii) 322 lotteries with mean around 3.5% (face value 100), and (iv) 19 lotteries with mean around 8% (face value 200). Note that the regression results are not sensitive to whether bond 1980 with the extreme lottery mean and variance is included. Without 1980, we get $a = 0.0080$ (s.e. = 0.0042), $b = 1.260$ (s.e. = 0.322) and $c = -0.182$ (s.e. = 0.081).

We have seen in Section 2.4 that bond 1977 has been traded at negative yields to maturity. Green and Rydqvist (1999) develop a model which explains that a tax arbitrage similar to the model we have studied here, combined with an effective short sales constraint, can raise the bond price to a level where yield to maturity gets negative. Their result requires that the marginal tax rate of the high-tax investors is reflected in the price drop over the lottery ($\gamma > 0$). Therefore, if we are right that the marginal tax rate is zero ($\gamma = 0$), their model cannot explain the negative yields to maturity in the Danish lottery bond market. This is a puzzle which we leave for future research.

4.4. Institutional Changes

Significant institutional changes took place in the middle of the sample period. In 1987, the professional traders shifted from personal to corporate tax, in July 1988, broker commission approximately doubled, and, in November 1988, the Copenhagen Stock Exchange introduced the electronic trading system. In this section, we report that the premium decreased after these changes and argue that this is consistent with an overall reduction of transaction costs.¹⁶

¹⁶ With the new trading system, we also change data sets from buy limit orders to weighted average transaction prices. Using buy limit orders from 1990 to 2000, but excluding 1989 for which we do not have such data, we find that the average premium falls from 1.135 using transaction prices to 1.091 using buy limit orders, so the change of data sets does not seem important.

TABLE X
Dummy Variable Regression

Intercept		$E(C)$		$V(C)$	t test	t test	R^2
a_0	a_1	b_0	b_1	c	$a_0 = a_1$	$b_0 = b_1$	
0.0195 (0.0051)	0.0090 (0.0060)	0.852 (0.334)	0.632 (0.352)	-0.118 (0.079)	1.98	0.77	0.081

Note. The price change is regressed on the lottery mean and variance with dummy variables for the time periods before and after 20 November 1988. Subindex 0 refers to the early period, and subindex 1 to the recent period. The coefficient of the variance is multiplied by 10,000. The parameters are estimated with ordinary least squares. Standard errors are reported below. The number of observations is 485.

We split the sample into observations before and after the introduction of the electronic trading system on 20 November 1988. The mean premium for each subperiod is reported in Table IX. In the top, we can see that the mean across all the bonds decreases from 1.798 to 1.135 with a t statistic of the difference equal to 3.1. Below, we can see that the cross-section pattern is the same in both periods. We do not report t statistics for the individual bonds, since the number of observations is small for each bond and subperiod.

Next, we reestimate the structural regression (5) with dummy variables for the two subperiods. Given the interpretation of the intercept as the marginal, round-trip transaction cost, it may increase or decrease. The increase in the commission schedule should raise the intercept, while the introduction of the electronic trading should lower it. The slope coefficient in the regression (5) is interpreted as the marginal valuation of the coupon lottery. If our conclusion from Section 4.3 above is right, that the professional traders' demand for tax shields is exhausted ($\gamma = 0$), the slope coefficient should be equal to one in both subperiods.

The regression results are reported in Table X. We can see that the intercept is significantly different in the two subperiods and decreases from 1.95 to 0.90%. The slope coefficient of the lottery mean is close to one in both periods and not statistically different from each other. These estimates suggest that transaction costs have decreased over time, but the standard errors are very large, so we feel uncomfortable making further inferences on these results.

5. CONCLUSION

We have studied the ex-day behavior of security prices in a market where professional traders prefer the cash distribution and individual investors must take the other side of the trade. These roles are the reverse of those in the U.S. equity market. As a result, we should observe that prices fall by *more* than the value of the cash distribution. Furthermore, if Elton and Gruber (1970) are right, we should find that the marginal valuation of the coupon lottery exceeds one while, if Boyd and Jagannathan (1994) are right, that the marginal valuation should be equal to

one. Consistent with both models, we find that prices fall on average by more than the cash payment, but consistent only with Boyd and Jagannathan (1994), that the marginal valuation is insignificantly different from one. Hence, our results are consistent with the predictions of the costly arbitrage model which has been offered as an explanation for the ex-day return behavior of U.S. stocks.

The standard errors are large so we cannot safely determine that the tax-driven demand for the coupon lottery is exhausted. However, we can reject the hypothesis that the professional traders compete away the profit from the tax arbitrage ($\gamma = 1$). Ruling out this particular case is interesting, because it supports Kalay's (1982) intuition that clean marginal tax rates cannot be imputed from ex-day returns when there is more than one tax clientele in the market. This seems to be true also in our case.

We have also found that investors in this market do not like the lottery risk and command a risk premium to bear this risk. Ex-day returns and current yields increase monotonically with the lottery variance. This suggests that the issuer has designed securities which the market does not want. As we have shown, new lottery bonds have been possible to sell only at coupon rates which exceed the current yields of traded securities. This is a warning to governments which plan to issue lottery bonds.

Our paper is a case study of the price formation in a market with two tax clienteles where one party (the professional traders) gets the entire surplus from trade. However, the extreme split of the gain from trade does not always have to be the case as suggested by the evidence in McDonald (2001). Future research should try to identify in which situations one party takes it all and when the parties reach an interior split of the gain from trade. This could tell us about the restrictions which are necessary for an equilibrium to exist with multiple tax clienteles (Dammon and Green (1987)). We may also learn about the details of the price formation process which leads to the competitive equilibrium (Rubinstein and Wolinsky (1985)).

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